



Options on Stock Indices and Currencies

Chapter 15

Index Options (page 325-335)

- The most popular underlying indices in the U.S. are
 - The S&P 100 Index (OEX and XEO)
 - The S&P 500 Index (SPX)
 - The Dow Jones Index times 0.01 (DJX)
 - The Nasdaq 100 Index
- Contracts are on 100 times index; they are settled in cash; OEX is American; the XEO and all other are European

Index Option Example

- Consider a call option on an index with a strike price of 900
- Suppose 1 contract is exercised when the index level is 880
- What is the payoff?

Using Index Options for Portfolio Insurance

- Suppose the value of the index is S_0 and the strike price is K
- If a portfolio has a β of 1.0, the portfolio insurance is obtained by buying 1 put option contract on the index for each $100S_0$ dollars held
- If the β is not 1.0, the portfolio manager buys β put options for each $100S_0$ dollars held
- In both cases, K is chosen to give the appropriate insurance level

Example 1

- Portfolio has a beta of 1.0
- It is currently worth \$500,000
- The index currently stands at 1000
- What trade is necessary to provide insurance against the portfolio value falling below \$450,000?

Example 2

- Portfolio has a beta of 2.0
- It is currently worth \$500,000 and index stands at 1000
- The risk-free rate is 12% per annum
- The dividend yield on both the portfolio and the index is 4%
- How many put option contracts should be purchased for portfolio insurance?

Calculating Relation Between Index Level and Portfolio Value in 3 months

- If index rises to 1040, it provides a $40/1000$ or 4% return in 3 months
- Total return (incl. dividends)=5%
- Excess return over risk-free rate=2%
- Excess return for portfolio=4%
- Increase in Portfolio Value= $4+3-1=6\%$
- Portfolio value=\$530,000

Determining the Strike Price

(Table 15.2, page 327)

Value of Index in 3 months	Expected Portfolio Value in 3 months (\$)
1,080	570,000
1,040	530,000
1,000	490,000
960	450,000
920	410,000

An option with a strike price of 960 will provide protection against a 10% decline in the portfolio value

Currency Options

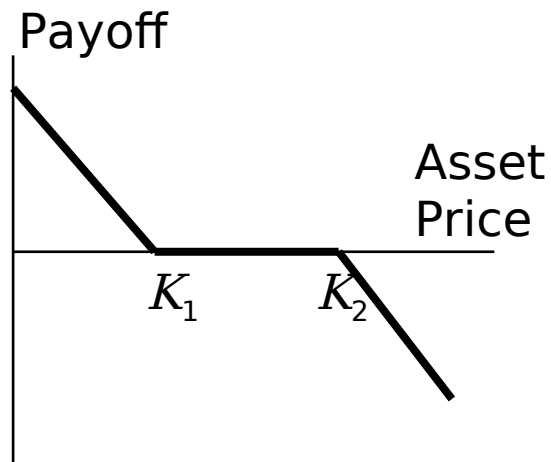
- Currency options trade on the Philadelphia Exchange (PHLX)
- There also exists a very active over-the-counter (OTC) market
- Currency options are used by corporations to buy insurance when they have an FX exposure

Range Forward Contracts

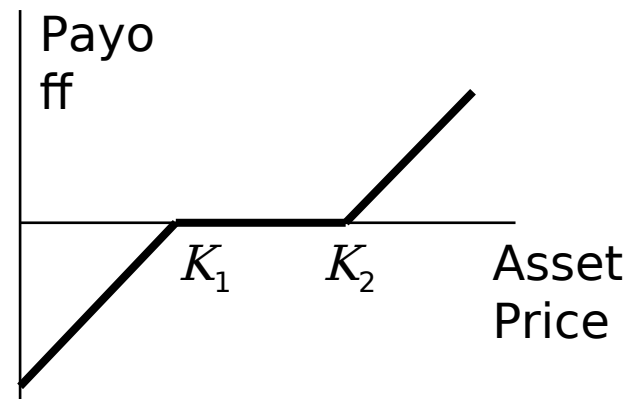
- Have the effect of ensuring that the exchange rate paid or received will lie within a certain range
- When currency is to be paid it involves selling a put with strike K_1 and buying a call with strike K_2 (with $K_2 > K_1$)
- When currency is to be received it involves buying a put with strike K_1 and selling a call with strike K_2
- Normally the price of the put equals the price of the call

Range Forward Contract

continued Figure 15.1, page 328



Short
Position



Long
Position

European Options on Stocks Providing a Dividend Yield

We get the same probability distribution for the stock price at time T in each of the following cases:

1. The stock starts at price S_0 and provides a dividend yield $= q$
2. The stock starts at price $S_0 e^{-qT}$ and provides no income

European Options on Stocks

Providing Dividend Yield

continued

We can value European options by reducing the stock price to $S_0 e^{-qT}$ and then behaving as though there is no dividend

Extension of Chapter 9 Results

(Equations 15.1 to 15.3)

Lower Bound for calls:

$$c \geq S_0 e^{-qT} - K e^{-rT}$$

Lower Bound for
puts

$$p \geq K e^{-rT} - S_0 e^{-qT}$$

Put Call Parity

$$c + K e^{-rT} = p + S_0 e^{-qT}$$

Extension of Chapter 13 Results

(Equations 15.4 and 15.5)

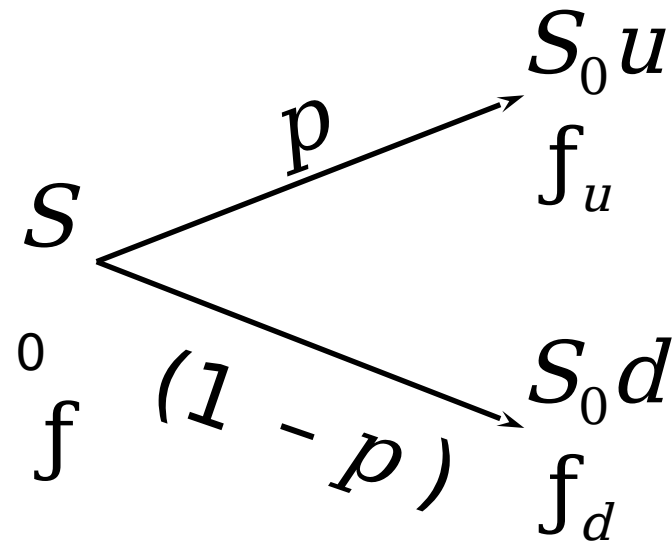
$$c = S_0 e^{qT} N(d_1) - K e^{rT} N(d_2)$$

$$p = K e^{rT} N(-d_2) - S_0 e^{qT} N(-d_1)$$

$$\text{where } d_1 = \frac{\ln(S_0 / K) + (r - q + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - q - \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

The Binomial Model



$$f = e^{-rT} [pf_u + (1-p)f_d]$$

The Binomial Model

continued

- In a risk-neutral world the stock price grows at $r-q$ rather than at r when there is a dividend yield at rate q
- The probability, p , of an up movement must therefore satisfy

$$pS_0u + (1-p)S_0d = S_0e^{(r-q)T}$$

so that

$$p = \frac{e^{(r-q)T} - d}{u - d}$$

The Foreign Interest Rate

- We denote the foreign interest rate by r_f
- When a U.S. company buys one unit of the foreign currency it has an investment of S_0 dollars
- The return from investing at the foreign rate is $r_f S_0$ dollars
- This shows that the foreign currency provides a “dividend yield” at rate r_f

Valuing European Index Options

We can use the formula for an option on a stock paying a dividend yield

Set S_0 = current index level

Set q = average dividend yield expected during the life of the option

Alternative Formulas (page 333)

$$c = e^{-rT} [F_0 N(d_1) - KN(d_2)]$$

$$p = e^{-rT} [KN(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

where

$$F_0 = S_0 e^{(r - q)T}$$

Valuing European Currency Options

- A foreign currency is an asset that provides a “dividend yield” equal to r_f
- We can use the formula for an option on a stock paying a dividend yield :

Set S_0 = current exchange rate

Set $q = r_f$

Formulas for European Currency Options

(Equations 15.11 and 15.12, page 333)

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-r T} N(d_2)$$

$$p = K e^{-r T} N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

$$\text{where } d_1 = \frac{\ln(S_0 / K) + (r - r_f + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - r_f - \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

Alternative Formulas

(Equations 15.13 and 15.14, page 322)

Using $F_0 = S_0 e^{(r - r_f)T}$

$$c = e^{-rT} [F_0 N(d_1) - KN(d_2)]$$

$$p = e^{-rT} [KN(-d_2) - F_0 N(-d_1)]$$

$$d_1 = \frac{\ln(F_0 / K) + \sigma^2 T / 2}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$